2.7 Solving Problems Involving More than One Right Triangle

**FOCUS** Use trigonometric ratios to solve problems that involve more than one right triangle.

When a problem involves more than one right triangle, we can use information from one triangle to solve the other triangle.

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**Example 1** Solving a Problem with Two Triangles

Find the length of BC to the nearest tenth of a centimetre.

![Diagram of triangle ABD with angles and sides labeled]

To solve a right triangle we must know:
- the lengths of two sides, or
- the length of one side and the measure of one acute angle

**Solution**

First use \( \triangle ABD \) to find the length of BD.

\[
\sin A = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin A = \frac{BD}{AB}
\]

\[
\sin 26^\circ = \frac{BD}{22.9}
\]

\[
22.9 \sin 26^\circ = BD
\]

\[
BD = 10.0386\ldots
\]

Do not clear the calculator screen.

In \( \triangle BCD \), find the length of BC.

\[
\sin C = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin C = \frac{BD}{BC}
\]

\[
\sin 49^\circ = \frac{10.0386\ldots}{BC}
\]

\[
BC \sin 49^\circ = 10.0386\ldots
\]

\[
BC = \frac{10.0386\ldots}{\sin 49^\circ}
\]

\[
BC = 13.3014\ldots
\]

BC is about 13.3 cm long.
1. Find the measure of $\angle F$ to the nearest degree.

Use $\triangle DEG$ to find the length of $EG$. Use the sine ratio.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D =$$

$$\sin ____ =$$

$$EG =$$

In $\triangle EFG$, use the ______ ratio to find $\angle F$.

The measure of $\angle F$ is about ______.

The angle of elevation is the angle between the horizontal and a person's line of sight to an object above.
Example 2  Solving a Problem Involving Angle of Elevation

Jason is lying on the ground midway between two trees, 100 m apart. The angles of elevation of the tops of the trees are 13° and 18°. How much taller is one tree than the other? Give the answer to the nearest tenth of a metre.

Solution

Jason is midway between the trees. So, the distance from Jason to the base of each tree is: \( \frac{100\, m}{2} = 50\, m \)

Use \( \triangle JKM \) to find the length of JK.

We know \( \angle M = 13° \).
JK is opposite \( \angle M \).
JM is adjacent to \( \angle M \).
Use the tangent ratio.

\[
\tan M = \frac{\text{opposite}}{\text{adjacent}} = \frac{JK}{JM}
\]

Substitute: \( \angle M = 13° \) and JM = 50

\[
tan 13° = \frac{JK}{50}
\]

\[
50 \tan 13° = JK
\]

JK = 11.5434…

Use \( \triangle MNP \) to find the length of NP.

We know \( \angle M = 18° \).
NP is opposite \( \angle M \).
MP is adjacent to \( \angle M \).
Use the tangent ratio.

\[
\tan M = \frac{\text{opposite}}{\text{adjacent}} = \frac{NP}{MP}
\]

Substitute: \( \angle M = 18° \) and MP = 50

\[
tan 18° = \frac{NP}{50}
\]

\[
50 \tan 18° = NP
\]

NP = 16.2459…

To find how much taller one tree is than the other, subtract:

16.2459… m – 11.5434… m = 4.7025… m

One tree is about 4.7 m taller than the other.
1. The angle of elevation of the top of a tree, T, is 27°. From the same point on the ground, the angle of elevation of a hawk, H, flying directly above the tree is 43°. The tree is 12.7 m tall. How high is the hawk above the ground? Give your answer to the nearest tenth of a metre.

We want to find the length of HG. Use \( \triangle QTG \) to find the length of QG. Use the tangent ratio.

\[
\tan Q = \frac{QG}{GT}
\]

Substitute: \( \tan 27° \) and \( \tan 43° \)

\[
\tan _____ = ________
\]

\[
QG = ________
\]

In \( \triangle QHG \), use the tangent ratio to find HG.

\[
HG = ________
\]

The hawk is about ________ above the ground.

The **angle of depression** is the angle between the horizontal and a person’s line of sight to an object below.
From a small plane, $V$, the angle of depression of a sailboat is $21^\circ$.
The angle of depression of a ferry on the other side of the plane is $52^\circ$.
The plane is flying at an altitude of 1650 m.
How far apart are the boats, to the nearest metre?

**Solution**

We want to find the length of $UW$.
The angle of depression of the sailboat is $21^\circ$.
So, in $\triangle UVX$, $\angle V = 90^\circ - 21^\circ$, or $69^\circ$.

Use $\triangle UVX$ to find the length of $UX$.

\[
\tan V = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan V = \frac{UX}{VX}
\]

\[
\tan 69^\circ = \frac{UX}{1650}
\]

$1650 \tan 69^\circ = UX$

$UX = 4298.3969\ldots$

The angle of depression of the ferry is $52^\circ$.
So, $\angle V$ in $\triangle VWX$ is: $90^\circ - 52^\circ$, or $38^\circ$.

Use $\triangle VWX$ to find the length of $WX$.

\[
\tan V = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan V = \frac{WX}{VX}
\]

\[
\tan 38^\circ = \frac{WX}{1650}
\]

$1650 \tan 38^\circ = WX$

$WX = 1289.1212\ldots$

To find the distance between the boats, add:
$4298.3969\ldots \text{ m} + 1289.1212\ldots \text{ m} = 5587.5182\ldots \text{ m}$

The boats are about 5588 m apart.
Check

1. This diagram shows a falcon, F, on a tree, with a squirrel, S, and a chipmunk, C, on the ground. From the falcon, the angles of depression of the animals are 36° and 47°. How far apart are the animals on the ground to the nearest tenth of a metre?

We want to find the length of CS.

\[ CS = GS - GC \]

The angle of depression of the squirrel is _____.

So, \( \angle F \) in \( \triangle FSG \) is: \( 90^\circ - _____ \), or _____.

Use \( \triangle FSG \) to find the length of GS.

\[ \tan ____ = \frac{_____}{_____} \]

\[ \tan ____ = \frac{_____}{_____} \]

\[ \tan ____ = \frac{_____}{_____} \]

\[ GS = \frac{_____}{_____} \]

The angle of depression of the chipmunk is _____.

So, \( \angle F \) in \( \triangle FCG \) is: \( 90^\circ - _____ \), or _____.

Use \( \triangle FCG \) to find the length of GC.

\[ GC = \frac{_____}{_____} \]

To find the distance between the animals, subtract:

\( \frac{_____}{_____} - \frac{_____}{_____} = \frac{_____}{_____} \)

The animals on the ground are about _____ apart.
1. Find the measure of $\angle C$ to the nearest degree.

   Use $\triangle ABD$ to find the length of $BD$.

   Use the tangent ratio.

   $\tan A = \frac{\text{opp}}{\text{adj}}$

   $\tan A = \frac{\text{opp}}{\text{adj}}$

   $\tan ____ = ____$

   $BD = \text{opp}$

   In $\triangle BCD$, use the _____ ratio to find $\angle C$.

   The measure of $\angle C$ is about ________.
2. Two guy wires support a flagpole, FH. The first wire is 11.2 m long and has an angle of inclination of $39^\circ$. The second wire has an angle of inclination of $47^\circ$. How tall is the flagpole to the nearest tenth of a metre?

We want to find the length of FH. Use \( \triangle EGH \) to find the length of EH. Use the cosine ratio.

\[
\cos E = \frac{EH}{EH}
\]

\[
\cos E = \frac{11.2}{EH}
\]

\[
\cos \theta = \frac{EH}{11.2}
\]

\[
EH = \frac{11.2}{\cos \theta}
\]

In \( \triangle EFH \), use the ______ ratio to find the length of FH.

\[
FH = \frac{EH}{\cos \phi}
\]

The flagpole is about ______ tall.
3. A mountain climber is on top of a mountain that is 680 m high. The angles of depression of two points on opposite sides of the mountain are 48° and 32°. How long would a tunnel be that runs between the two points? Give your answer to the nearest metre.

We want to find the length of QN.

The angle of depression of point Q is _____.

So, \( \angle M \) in \( \triangle PQM \) is: \( 90^\circ - \), or _____.

Use \( \triangle PQM \) to find the length of PQ.

Use the ______ ratio.

\[
\text{PQ} = \text{___________}
\]

The angle of depression of point N is _____.

So, \( \angle M \) in \( \triangle PMN \) is: \( 90^\circ - \), or _____.

Use \( \triangle PMN \) to find the length of PN.

Use the ______ ratio.

\[
\text{NP} = \text{___________}
\]

The length of the tunnel is: _____ = _____ + _____

\[
\text{QN} = \text{___________}
\]

The tunnel would be about ________ long.